

# Making Ends Meet: String Unification and Low-Energy Data

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## Abstract

A long-standing problem in string phenomenology has been the fact that the string unification scale disagrees with the GUT scale obtained by extrapolating low-energy data within the framework of the minimal supersymmetric standard model (MSSM). In this paper we examine several effects that may modify the minimal string predictions and thereby bring string-scale unification into agreement with low-energy data. These include heavy string threshold corrections, non-standard hypercharge normalizations, light SUSY thresholds, intermediate gauge structure, and thresholds arising from extra matter beyond the MSSM. We explicitly evaluate these contributions within a variety of realistic free-fermionic string models, including the flipped  $SU(5)$ ,  $SO(6) \times SO(4)$ , and various  $SU(3) \times SU(2) \times U(1)$  models, and find that most of these sources do not substantially alter the minimal string predictions. Indeed, we find that the only way to reconcile string unification with low-energy data is through certain types of extra matter. Remarkably, however, many of the realistic string models contain precisely this required matter in their low-energy spectra.

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Heterotic string theories have a number of properties which make them the leading candidates for a unified theory of the fundamental particles and interactions. Not only do they provide a first-quantized description of gravity, for example, but they also incorporate  $N = 1$  supersymmetric gauge theories as their low-energy limits. This is an important feature, for such supersymmetric gauge theories are natural extensions of the Standard Model which are in agreement with all low-energy experimental data. Moreover, one such theory in particular, namely the minimal supersymmetric Standard Model (MSSM), provides a successful scenario for gauge coupling unification, with a predicted unification scale  $M_{\text{MSSM}} \approx 2 \times 10^{16}$  GeV. String theories, by contrast, predict gauge coupling unification at a somewhat larger unification scale, typically  $M_{\text{string}} \approx g_{\text{string}} \times 5 \times 10^{17}$  GeV where  $g_{\text{string}} \approx 0.8$  at unification. This discrepancy between these two unification scales implies that string theory naively predicts incorrect values for the electroweak and strong couplings  $\sin^2 \theta_W$  and  $\alpha_{\text{strong}}$  at the  $Z$  scale. Resolving this discrepancy and “making the two ends meet” is therefore one of the major problems confronting string phenomenology.

There are many possible effects which may account for this discrepancy and alter the running of the gauge couplings between the high and low energy scales. First, there are the so-called “heavy string threshold corrections” which represent the contributions from the massive Planck-scale string states that are otherwise neglected in an analysis of the purely low-energy (*i.e.*, massless) string spectrum. Second, there are potential corrections due to the fact that in string theory, the normalization of the  $U(1)$  hypercharge need not take the standard value that it has in various field-theoretic GUT models. Third, if supersymmetry is ultimately broken at the TeV-scale, the required SUSY-breaking terms will lead to additional light SUSY thresholds. Fourth, in various GUT scenarios, there can be corrections due to the presence of non-trivial gauge structure at intermediate scales. Finally, there may also be contributions from additional exotic states beyond those predicted by the MSSM. While such states are not expected in standard field-theoretic GUT scenarios, we shall see that they appear naturally in certain self-consistent string models.

Many of these effects have been discussed previously, each in an abstract setting and in isolation. However, within the tight constraints of a given realistic string model, the mechanisms giving rise to three generations and the MSSM gauge group may prove inconsistent with, for example, large threshold corrections or extra non-MSSM matter. Moreover, the increased complexity of the known realistic string models may substantially alter previous expectations based on simplified or idealized scenarios. It is therefore important to rigorously calculate all of these effects simultaneously, within the context of a wide variety of actual realistic string models, in order to determine which path to unification (if any) such models actually take.

In this paper, we shall present the results of the first such systematic evaluation. A more complete discussion of our analysis and results will be presented in Ref. [1]. Surprisingly, we find that most of these effects in these models cannot resolve the discrepancy between string unification and low-energy data. In particular, they do

not modify the one-loop renormalization group equations (RGE's) in a manner that would yield the correct values of  $\alpha_{\text{strong}}$  and  $\sin^2 \theta_W$  at the  $Z$ -scale when starting at the string scale. Indeed, we find that only the effect of certain extra exotic states has the potential to bring the low-energy data into agreement with string unification. Remarkably, these states, which take the form of extra color triplets and electroweak doublets with special  $U(1)$  quantum numbers, appear naturally in the realistic free-fermionic string models. Thus string theory, which predicts an unexpectedly high unification scale  $M_{\text{string}}$ , in many cases also simultaneously predicts precisely the extra exotic particles needed to reconcile this higher scale with low-energy data.

The string models that we have chosen for our analysis include the flipped  $SU(5)$  model of Ref. [2], the  $SO(6) \times SO(4)$  model of Ref. [3], and various string models [4, 5] in which the Standard-Model gauge group  $SU(3) \times SU(2) \times U(1)$  is realized directly at the Planck scale. As required, all of these models have  $N = 1$  spacetime supersymmetry, and contain exactly three generations in their massless spectra. Moreover, these models also naturally incorporate the fermion mass hierarchy with a heavy top-quark mass. These models can all be realized in the free-fermionic construction [6], and their phenomenological properties are ultimately a consequence of their common underlying  $Z_2 \times Z_2$  orbifold structure [7]. In the free-fermionic construction, this structure is realized through the so-called “NAHE set” of boundary-condition basis vectors.

String theory predicts in general that at tree level, the gauge couplings  $g_i$  corresponding to each gauge group factor  $G_i$  realized at Kač-Moody level  $k_i$  will unify with the gravitational coupling constant  $G_N$  according to [8]

$$g_{\text{string}}^2 = 8\pi \frac{G_N}{\alpha'} = g_i^2 k_i \quad \text{for all } i \quad (1)$$

where  $\alpha'$  is the Regge slope. At the one-loop level, however, these relations are modified to

$$\frac{16\pi^2}{g_i^2(\mu)} = k_i \frac{16\pi^2}{g_{\text{string}}^2} + b_i \ln \frac{M_{\text{string}}^2}{\mu^2} + \Delta_i^{(\text{total})} \quad (2)$$

where  $b_i$  are the one-loop beta-function coefficients, and where the  $\Delta_i^{(\text{total})}$  represent the combined corrections from each of the effects discussed above. Thus, in a given realistic string model, we can use Eq. (2) to find the expected values of the strong and electroweak gauge couplings at the  $Z$ -scale, and thereby obtain explicit expressions for  $\alpha_{\text{strong}}(M_Z)$  and  $\sin^2 \theta_W(M_Z)$ .

We organize the calculation as follows. In each case we take  $\alpha_{\text{e.m.}}(M_Z) = 1/127.9$  as a fixed input parameter, and leave  $k_1$  arbitrary. We then solve Eqs. (2) for  $i = 1, 2, 3$  simultaneously in order to eliminate the direct dependence on  $g_{\text{string}}$  from the first term on the right sides of Eqs. (2). A small dependence on  $g_{\text{string}}$  remains through  $M_{\text{string}}$ , and in all subsequent numerical calculations we will allow  $M_{\text{string}}$  to vary in the range  $(2 - 7) \times 10^{17}$  GeV in order to account for this. In each case we initially assume the MSSM spectrum between the Planck scale and the  $Z$ -scale, and treat all

perturbations of this scenario through effective correction terms. The expression for  $\alpha_{\text{strong}}(M_Z)$  then takes the general form

$$\begin{aligned} \alpha_{\text{strong}}^{-1}(M_Z) = & \Delta_{\text{MSSM}}^{(\alpha)} + \Delta_{\text{h.s.}}^{(\alpha)} + \Delta_{\text{l.s.}}^{(\alpha)} \\ & + \Delta_{\text{i.g.}}^{(\alpha)} + \Delta_{\text{i.m.}}^{(\alpha)} + \Delta_{\text{other}}^{(\alpha)}, \end{aligned} \quad (3)$$

and likewise for  $\sin^2 \theta_W(M_Z)$  with corresponding corrections  $\Delta^{(\sin)}$ . Here  $\Delta_{\text{MSSM}}$  represents the one-loop contributions from the MSSM spectrum alone, and the remaining  $\Delta$  terms respectively correspond to heavy string thresholds, light SUSY thresholds, intermediate-scale gauge structure, and intermediate-scale extra matter. Each of these  $\Delta$  terms has an algebraic expression in terms of  $\alpha_{\text{e.m.}}$  as well as model-specific parameters such as  $k_1$ , the beta-function coefficients, and any appropriate intermediate mass scales. Finally, we also include in our analysis various two-loop and Yukawa-coupling effects, as well as explicit correction factors arising from scheme conversion (from the SUSY-based  $\overline{DR}$  scheme to the  $\overline{MS}$  scheme relevant for comparisons with low-energy data). These are represented by  $\Delta_{\text{other}}$ .

We have explicitly calculated each of these  $\Delta$  contributions for each of the realistic string models discussed above. Our results are as follows.

Two-loop, Yukawa, and scheme conversion: We first focus on these “other” corrections. As discussed above, these are calculated assuming the MSSM spectrum between the string unification and  $Z$  scales. To estimate the size of the second-loop corrections, we run the one- and two-loop RGE’s for the gauge couplings and take the difference. Likewise, to estimate the Yukawa-coupling corrections, we evolve the two-loop RGE’s for the gauge couplings coupled with the one-loop RGE’s for the heaviest-generation Yukawa couplings, assuming  $\lambda_t \approx 1$  and  $\lambda_b = \lambda_\tau \approx 1/8$  at the string unification scale. We then subtract the two-loop non-coupled result. As indicated above, these differences are then each averaged for different values of  $M_{\text{string}}$ . Including the standard scheme-conversion corrections as well, we find

$$\Delta_{\text{other}}^{(\sin)} \approx 6.13 \times 10^{-3}, \quad \Delta_{\text{other}}^{(\alpha)} \approx 0.700. \quad (4)$$

Non-standard hypercharge normalizations: We have studied the effect of varying the value of  $k_1$  within the RGE’s, as proposed, for example, in Ref. [9]. Although  $k_1 = 5/3$  in standard  $SO(10)$  GUT scenarios, in string theory the hypercharge normalizations can generally be different. We find, however, that the experimental discrepancies are resolved only for values  $k_1 \approx 1.4$ , whereas each of these string models ultimately turns out to have  $k_1 \geq 5/3$ . Thus, this effect cannot resolve the discrepancy between the GUT and string scales in these models. Moreover, arguments exist [10] showing that  $k_1 \geq 5/3$  in any free-field heterotic string model containing the MSSM gauge group and spectrum realized at level  $k_2 = k_3 = 1$ . This constraint arises because  $k_1$  is related to the embedding of the hypercharge in terms of simple worldsheet  $U(1)$  currents with proper conformal-field-theory normalizations. One then finds [10] that the minimal embedding yielding the correct hypercharges for

MSSM particles corresponds to  $k_1 = 5/3$ . Thus, it is not likely that other level-one models can exploit this effect either.

Heavy string threshold corrections: Calculating the heavy string threshold corrections within the realistic free-fermionic models is by far the most complex part of the analysis, for there are numerous subtleties arising due to twisted fermionic boundary conditions and highly non-trivial free-fermionic realizations of the gauge groups due to occasional enhanced gauge symmetries. A full account of our analysis will be presented in Ref. [1]. We follow Kaplunovsky [11] in defining the threshold corrections  $\Delta_i$  corresponding to each gauge-group factor  $G_i$  as

$$\Delta_i \equiv \int_{\mathcal{F}} \frac{d^2\tau}{\text{Im } \tau} \left\{ \text{Tr } \overline{Q}_h^2 Q_i^2 q^H \overline{q}^{\overline{H}} - b_i \right\} \quad (5)$$

where  $\overline{Q}_h$  is the spacetime helicity operator,  $Q_i$  is the internal gauge charge operator for the group  $G_i$ ,  $b_i$  is the corresponding one-loop beta-function coefficient, and  $H$  and  $\overline{H}$  are respectively the left- and right-moving worldsheet Hamiltonians. The trace is then performed over all sectors in the theory and all mass levels in each sector, and the result with  $b_i$  subtracted is then integrated over the fundamental domain  $\mathcal{F}$  of the modular group. In the realistic models we consider, there are typically thousands of sectors which make non-vanishing contributions to  $\Delta_i$ , and great care has to be taken with regards to the GSO projection phases in order to verify that the desired spectrum is produced. Other subtleties include deriving explicit expressions for  $Q_i^2$  in terms of the individual charges  $Q_L$  corresponding to each worldsheet fermion; in the cases of string models with enhanced gauge symmetries, this embedding of the gauge charges within the free fermions can be highly non-trivial. Finally, and equally importantly, one must perform the integrations in Eq. (5) in such a way that numerical errors are minimized and all logarithmic divergences cancelled. Details can be found in Ref. [1]. Note that because these are  $(2, 0)$  models [rather than  $(2, 2)$ ], one cannot use various moduli-dependent approximations found in the literature.

The various quantities  $\Delta_i$  one obtains must then be appropriately combined in order to yield expressions for  $\Delta_{\text{h.s.}}^{(\alpha, \text{sin})}$  in Eq. (3). In particular, we find

$$\begin{aligned} \Delta_{\text{h.s.}}^{(\text{sin})} &= \frac{1}{2\pi} \frac{k_1}{k_1 + 1} \alpha_{\text{e.m.}} \frac{\Delta_2 - \Delta_{\hat{Y}}}{2} \\ \Delta_{\text{h.s.}}^{(\alpha)} &= \frac{-1}{4\pi} \frac{1}{k_1 + 1} [k_1(\Delta_{\hat{Y}} - \Delta_3) + (\Delta_2 - \Delta_3)] \end{aligned} \quad (6)$$

where  $\Delta_{3,2,\hat{Y}}$  respectively correspond to the strong, electroweak, and properly normalized weak hypercharge groups. It is important that only the relative *differences* of these quantities appear, for the definition in Eq. (5) neglects certain group-independent additive factors.

Our results are then as follows. For the  $SU(3) \times SU(2) \times U(1)_{\hat{Y}}$  model of Ref. [4], we find

$$\Delta_{\hat{Y}} - \Delta_3 = 5.0483, \quad \Delta_{\hat{Y}} - \Delta_2 = 1.6137, \quad (7)$$

so that  $\Delta_{\text{h.s.}}^{(\text{sin})} = -6 \times 10^{-4}$  and  $\Delta_{\text{h.s.}}^{(\alpha)} = -0.3536$ . Since the hypercharge operator in this model has the standard  $SO(10)$  embedding, we have taken  $k_1 = 5/3$ . Unfortunately, we see from the signs and sizes of these results that they effectively *increase* the string unification scale slightly, and thereby enhance the disagreement with experiment in this model.

In the remaining models, the Standard Model gauge group is realized only after the Planck-scale gauge group is broken at an intermediate scale  $M_I$ . We shall analyze the effects of such intermediate scales below, and thus assume for the purposes of this analysis that  $M_I \approx M_{\text{string}}$ . For the flipped  $SU(5) \times U(1)$  model of Ref. [2], we then find  $\Delta_1 - \Delta_5 = 7.681$ , so that after combining these results in a manner appropriate for the flipped  $SU(5)$  symmetry-breaking scenario [which for  $M_I = M_{\text{string}}$  amounts to taking  $\Delta_2 = \Delta_3 = \Delta_5$  and  $\Delta_{\hat{Y}} = (\Delta_5 + 24\Delta_1)/25$ ], we obtain the final low-energy corrections  $\Delta_{\text{h.s.}}^{(\text{sin})} = -2 \times 10^{-3}$  and  $\Delta_{\text{h.s.}}^{(\alpha)} = -0.29$ . This again implies a slight effective increase in the string unification scale.\* Likewise, for the  $SO(6) \times SO(4) \simeq SU(4) \times SU(2)_L \times SU(2)_R$  model of Ref. [3], we find

$$\Delta_{2_L} - \Delta_4 = 8.4763, \quad \Delta_{2_R} - \Delta_4 = 2.0483, \quad (8)$$

so that, after the Pati-Salam symmetry-breaking scenario [which for  $M_I = M_{\text{string}}$  implies that we take  $\Delta_3 = \Delta_4$ ,  $\Delta_2 = \Delta_{2_L}$ , and  $\Delta_{\hat{Y}} = (2\Delta_4 + 3\Delta_{2_R})/5$ ], we obtain the low-energy results  $\Delta_{\text{h.s.}}^{(\text{sin})} = -2.8 \times 10^{-3}$  and  $\Delta_{\text{h.s.}}^{(\alpha)} = -0.3141$ . This too implies a slight effective increase in the string unification scale. Finally, for the  $SU(3) \times SU(2) \times U(1)$  model of Ref. [5], we similarly find after a complicated pattern of symmetry-breaking that  $\Delta_{\text{h.s.}}^{(\text{sin})} = -1.2 \times 10^{-3}$  and  $\Delta_{\text{h.s.}}^{(\alpha)} = -0.335$ , again implying that  $M_{\text{string}}$  is effectively increased.

It is an important observation that the sizes of the threshold corrections are very small in all of these realistic string models, and thus do not greatly affect (either positively or negatively) the magnitude of the string unification scale. Thus, we conclude that these threshold corrections cannot by themselves resolve the experimental discrepancy. Moreover, despite the fact that such threshold corrections receive contributions from infinite towers of massive string states, we have been able to provide a general model-independent argument [1] which explains why these corrections must always be naturally suppressed in string theory (except of course for large moduli). This suppression arises due to the modular properties of the integrand of Eq. (5), in particular the insertion of the charge and helicity operators into the trace. These operators increase the modular weight of the integrand, and thereby suppress the value of the integral in tachyon-free models. Further details can be found in Ref. [1]. Hence alternative string models are not likely to have significantly larger threshold corrections.

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\* Note that our result for  $\Delta_1 - \Delta_5$  in this model disagrees by a factor of approximately three with that obtained in Ref. [12]. The size of our result, however, is more in line with those from the other realistic models we examine, as well as from previous heavy threshold calculations in various orbifold and Type-II models [13].

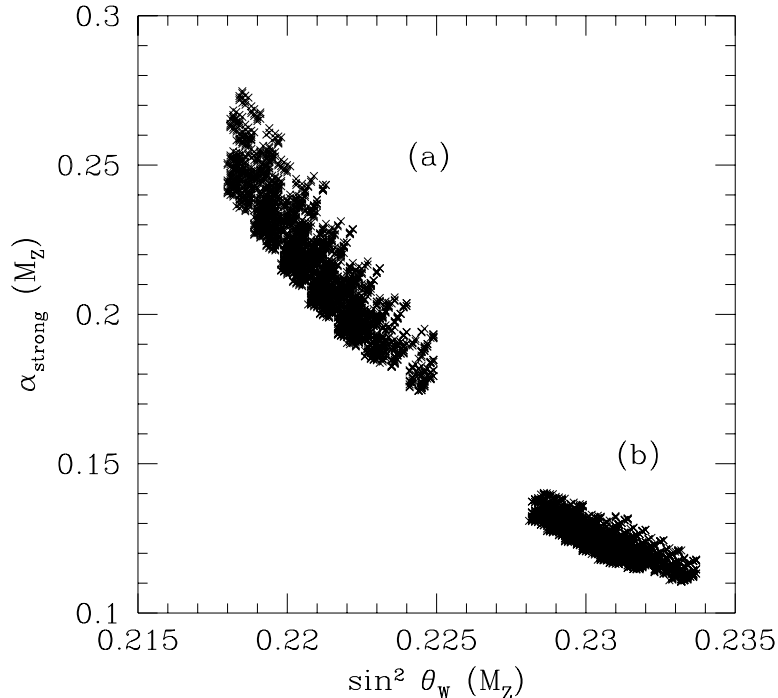


Figure 1: Scatter plot of  $\{\sin^2 \theta_W(M_Z), \alpha_{\text{strong}}(M_Z)\}$  for various values of  $\{m_0, m_{1/2}, m_h, m_{\tilde{h}}, M_{\text{string}}\}$ . Region (a) assumes the MSSM spectrum as discussed in the text, while (b) also includes the effects of the string-predicted extra matter.

Intermediate gauge structure: In the above calculations, we have implicitly assumed that  $M_I \approx M_{\text{string}}$ . In general, however, we will have  $M_I \leq M_{\text{string}}$ , and this has the potential to change the analysis. We have investigated this possibility, but find that in the above models, taking values  $M_I < M_{\text{string}}$  surprisingly only *enhances* the disagreement with experiment. Thus, once again, this cannot eliminate the experimental discrepancy.

Light SUSY thresholds: As a perturbation on the assumption of the MSSM spectrum from the string scale to the  $Z$  scale, one can also consider the effects of the light SUSY thresholds that arise from SUSY-breaking. These effects can ultimately be parametrized in terms of the four soft SUSY-breaking parameters  $\{m_0, m_{1/2}, m_h, m_{\tilde{h}}\}$ , assuming either universal or non-universal boundary terms for the sparticle masses. We find, however, that even in the “best-case” scenario, these effects cannot resolve the discrepancy with low-energy data. In Fig. 1(a) we have plotted the values of  $\{\sin^2 \theta_W(M_Z), \alpha_{\text{strong}}(M_Z)\}$  obtainable under variations of  $M_{\text{string}}$  and these SUSY-breaking parameters, assuming universal boundary conditions. It is clear that the experimentally measured values of  $\sin^2 \theta_W(M_Z) \approx 0.231$  and  $\alpha_{\text{strong}}(M_Z) \approx 0.125$  are not reached.

Extra string-predicted matter: Finally we consider the effects of extra string-predicted matter beyond the MSSM [5, 14]. While the introduction of such matter may seem *ad hoc* from the low-energy point of view, our analysis thus far indicates that this is the *only* way by which the experimental discrepancies might be reconciled in realistic free-fermionic models. Furthermore, such matter appears naturally in these models, and its effects must be included. This matter typically takes the form of additional color triplets and electroweak doublets, in vector-like representations. Note that the masses and  $U(1)$  quantum numbers of such states are highly model-dependent. The masses, in particular, can be calculated from cubic or higher non-renormalizable terms in the superpotential, and depend on the particular SUSY-breaking scheme employed. In general, however, these masses are often much smaller than the string scale. For example, in one model, the mass scale of such an additional color triplet was estimated [15] to be of the order of  $10^{11}$  GeV.

We have analyzed the effects that such matter can have on the unification of the couplings. Despite the presence of this new intermediate scale, however, we find that successful low-energy predictions are still generically difficult to obtain. Indeed, in order to accommodate the low-energy data, we find on general grounds that extra triplets *and* doublets must be present simultaneously, and moreover that this extra matter must have particular hypercharge assignments so as to modify the running of the strong and electroweak couplings without substantially affecting the  $U(1)$  coupling. These include, for example,  $SU(3) \times SU(2)$  representations such as  $(3, 2)_{1/6}$ ,  $(\bar{3}, 1)_{1/3}$ ,  $(\bar{3}, 1)_{1/6}$ , and  $(1, 2)_0$ , all of which have large values of  $b_2$  and  $b_3$ , but relatively small values of  $b_1$ . Many of the realistic free-fermion models (*e.g.*, that in Ref. [4]) do not have such exotic representations and can actually be ruled out on this basis.

Remarkably, however, some of the other realistic string models predict extra matter with exactly the required hypercharge assignments and in exactly the proper combinations for successful string-scale unification. For example, the model of Ref. [5] contains in its spectrum two pairs of  $(\bar{3}, 1)_{1/3}$  color triplets with beta-function coefficients  $(b_3, b_2, b_1) = (1/2, 0, 1/5)$ , one pair of  $(\bar{3}, 1)_{1/6}$  triplets with  $b_i = (1/2, 0, 1/20)$ , and three pairs of  $(1, 2)_0$  doublets with  $b_i = (0, 1/2, 0)$ . Clearly, this exotic matter cannot be fit into the standard  $SO(10)$  representations. Nevertheless, we find that this particular combination of representations and hypercharge assignments opens up a sizable window in which the low-energy data and string unification can be reconciled. For example, we find that if these triplets all have equal masses in the approximate range  $2 \times 10^{11} \leq M_3 \leq 7 \times 10^{13}$  GeV with the doublet masses in the corresponding range  $9 \times 10^{13} \leq M_2 \leq 7 \times 10^{14}$  GeV, then the discrepancy is removed. This situation is illustrated in Fig. 1(b), which plots the same points as in Fig. 1(a) except that this extra matter is now included in the analysis. Details and other scenarios for each of the realistic string models will be discussed in Ref. [1]. Of course, we emphasize that it is still necessary to verify that this extra matter can actually have the needed masses in these models.



We conclude, then, that string-scale unification places tight constraints on realistic free-fermion string models, and that only the appearance of extra exotic matter in particular representations can possibly resolve the experimental discrepancies. It is of course an old idea that the presence of extra matter can resolve the discrepancy between the GUT and string unification scales. What is highly non-trivial, however, is that this now appears to be the *only* way in which string theory can solve the problem. Furthermore, we have seen that a subset of these realistic models actually manage to satisfy these constraints in precisely this manner, naturally giving rise to the required sorts of extra states with the proper non-standard hypercharges to do the job. It will therefore be interesting to see whether this string-predicted extra matter has masses in the appropriate ranges, and to determine the effects that this extra matter might have on low-energy physics. Such work is in progress [16].

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